A QUESTION TO FRANK WILCZEK re Pythagorean-Comma/Golden-Section

Aubrey Meyer - 09 January 2016

In your really lovely book ***"A Beautiful Question"*** these observations about vibrational 'rate' (Hertz) relationships in music are made, as laid out by Pythagoras: -

> *"Pythagoras's first rule is a relationship between the length of the vibrating string and our perception of its tone. The rule says that two copies of the same type of string, both subject to the same tension, sound good together precisely when the length of the strings are in ratios of small whole numbers. Thus, for example, when the ratio is 1:2, the tones form an octave. When the ratio is 2:3 we hear the dominant fifth; when the ratio is 3:4, the major fourth. In musical notation (in the key of C), these correspond to playing two Cs, on above the other together, a C-G, or a C-F, respectively. People find those tone combinations appealing."*

Here is an inter-active animation demonstrating those rate-relationships audio-visually: - http://www.gci.org.uk/animations/vibrating-strings.swf

Your observations go a step further than Stephen Hawking in ***"The Grand Design"*** where he cites Pythagoras's discovery as the first instance of theoretical physics:

> *"According to legend, the first mathematical formulation of what we might today call a law of nature dates back to an Ionian named Pythagoras [who] is said to have discovered the numerical relationship between the length of the strings used in musical instruments and the harmonic combinations of the sounds. In today's language we would describe that relationship by saying that the frequency - the number of vibrations per second or 'Hertz' value - of a string vibrating under fixed tension is inversely proportional to the length of the string. One could call that simple mathematical formula the first instance of what we now know as the first law of theoretical physics."*

These are all beautiful observations that are surely linked to answering the beautiful question raised by your book. And while both books [a] reference material and demonstrate images that are rich in the legendary 'Golden Section' (GS 0.618 . . .) and also [b] that it can be argued that the Golden Section is Nature's smallest and most beautiful secret, neither book directly mentions it.

If we go further into numbers arising from Pythagoras's observations, it is apparent that while: -

A. 7 Perfect Octaves (2^7) and 12 Perfect 5ths (1.5^12) ***do not*** commute
B. 7 Perfect Octaves (2^7) and 12 Well-Tempered 5ths (1.49831^12) ***do*** commute
C. and that the gap between A & B is what has been called the 'Pythagorean Comma'.

Sometimes seen as a 'correction', this gives rise to the well-known Well-Tempered tuning or 'equal temperament'. This tuning system was adopted by J S Bach in his Well-Tempered Clavier, the 48 Preludes and Fugues. As it enables writing and performance in all keys, it has governed all Western Music since that time.

However, what appears not to have been recognized anywhere (yet) is this: -

Within this first law, as the Pythagorean Comma emerges it also creates a 'negative feedback' to the self-referential value of 'Phi' (0.618) or the 'Golden Section' (see overleaf). Do you have any thoughts about that? It seems so fundamental to the beautiful answer you give to the beautiful question you so boldly raise.

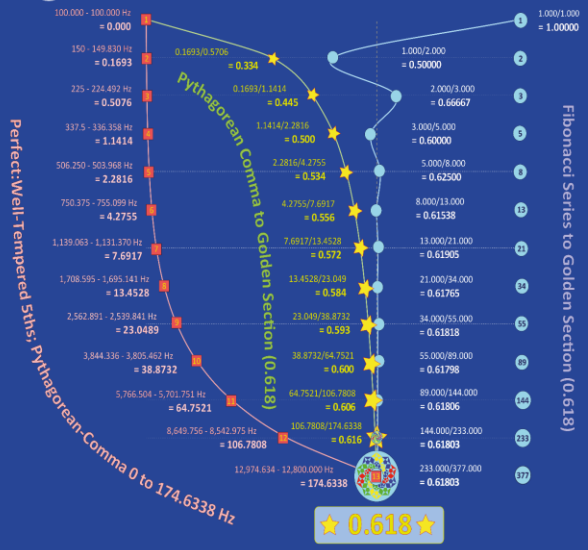
Nature's Smallest & Most Beautiful Secret

φ

The *'Golden Comma'*
Hz differences between
a cycle of Perfect 5ths &
Well Tempered 5ths
emerge & are known as
the 'Pythagorean Comma'.

These curve to φ as a
neg-entgropic feedback
(vide Schrodinger)
to the Comma.

Pythagorean Comma & Fibonacci Series converge on *'Golden ★ 0.618 ★ Section'*



	PERFECT OCTAVES 1*2	PERFECT 5THS 1*1.5	5THS WELL TEMPERED 1*1.498	PYTHAGOREAN COMMA Differential	COMMA EMERGES TO GOLDEN SECTION Feedback		FIBONACCI SERIES GOES TO GOLDEN SECTION Corroborates PC Feedback		
1	2.000	1.500	1.498	0.002	0.33352	This progression to the Golden Section (Phi) is completely beautiful	1	1.00000	1
2	4.000	2.250	2.245	0.005	0.44470		2	0.50000	2
3	8.000	3.375	3.364	0.011	0.50028		3	0.66667	3
4	16.000	5.063	5.040	0.023	0.53363		5	0.60000	4
5	32.000	7.594	7.551	0.043	0.55587		8	0.62500	5
6	64.000	11.391	11.314	0.077	0.57175		13	0.61538	6
7	**128.000**	17.086	16.951	0.135	0.58366		21	0.61905	7
8		25.629	25.398	0.230	0.59293		34	0.61765	8
9		38.443	38.055	0.389	0.60034		55	0.61818	9
10		57.665	57.018	0.648	0.60640		89	0.61798	10
11		86.498	85.430	1.068	0.61146		144	0.61806	11
12		**129.746**	**128.000**	1.746	0.61573	**0.618**	233	0.61803	12
13		194.620	191.783	2.836	0.61940		377	0.61804	13