

Concerning Musical Tuning, the Fibonacci Series and the Fine Structure Constant

Revised text 11/09/2017

If I were not a physicist, I would probably be a musician (Einstein)

The Pythagorean Comma is the discrepancy in pitch between the tuning of a note seven octaves above its fundamental by the traditional method of using perfect fifths of ratio 3:2, and its tuning by octaves of ratio 2:1. The arithmetical value of the Pythagorean Comma is $3^{12}/2^{19} = 531441/524288 = 1.013643\dots$. Since this discrepancy occurs when tuning the thirteenth note in the series, it could well have been the origin of the custom of treating thirteen as an unlucky number.

Throughout the history of music in the West, there have been three main systems of tuning. The earliest was Just tuning, based on regular numbers in the form $2^p 3^q$. These numbers appear in tables of both reciprocals and factorizations from Mesopotamia. Musicological evidence suggests that regular numbers may have been used for musical tuning since about 1800 BC. The second form of tuning is known as Pythagorean/Platonic. This uses numbers in the form $2^p 3^q 5^r$. As a result, the size of the semitone is decreased from 16:15 to 256:243. Finally, in modern times, since science has recognised that the octave can only be divided into twelve equal semitones when it is conceived as a logarithmic spiral in which each of the semitones is an increase in frequency by the twelfth root of 2. Such tuning is described as equal-tempered. Kappraff¹ uses a twelve-pointed star to illustrate an equal-tempered chromatic scale. The diameters of such a star then represent tritones (C-F#, D-G#): intervals which were known in earlier times as the devil in music (*diabolus in musica*).

The Fine Structure Constant is the strength of coupling of the electrical charge in a subatomic particle, expressed through the formula²: $\alpha = e^2/hc \sim 1/137$, (0.0072797351...) The only appearance of the prime number 137 which I have so far found which has a possible musical connection relates to the Golden Angle (137.5 degrees and its reciprocal). If the Golden Angle is drawn in both directions from the twelve-o'clock position on the circumference of the circle around Kappraff's twelve-pointed star, it would represent a point within the minor third (ratio 6:5) around each component of the tritone (e.g. for F#-C, between E and

¹ Kappraff, J., (2002) *Beyond Measure: a Guided Tour Through Nature, Myth, and Number* World Scientific, p.62, Fig 3.4 (d)

² Cf. The author's 'Fee-**phi**-fo-fum' on Academia.edu with regard to phyllotaxis and related matters

G, and between B and D). Merrick³, believes that the Fibonacci Series (i.e. 1, 1, 2, 3, 5, 8converging as it does on the Golden Ratio Φ (e.g. $610/377 = 1.618...$), serves as a natural damping proportion⁴. Maximum resonance occurs at 1.6, a major sixth (ratio 5:3); while the maximum damping corresponds to Φ (1.618). The distance between these two extremes has a ratio of about 7:12². Visually, the damping regions can be imagined as the space between the cracks of the white keys E-F and B-C on a modern keyboard .Furthermore, $\Phi^2/1 \times 6/5$ (the interval around the tritone) = π (3.14150).

Thus, in both science and in musical tuning by means of the Harmonic Series of overtones, there seems to be a series of essentially approximate relationships within or related to an intervallic space of a minor third (6:5), linking $\sqrt{2}$, π , Φ , α and a Pythagorean Comma⁵. Moreover, in actual performance, musicians taking account of both the resonances of their instrument and of the location, trust their ears, and thereby often deliberately depart from the expected theoretical tuning system. Would it be reasonable, then, to conclude that, despite the ambiguities surrounding irrational and transcendental numbers, there is an urgent need for physicists and cosmologists to focus their attention more seriously than they have so far done on Fibonacci patterning and the Harmonic Series overtones thereby generated? Einstein calculated that the speed of light in miles per second as 18682. The square root of this number is approximately 432. Interpreted as a tone-number, 432 defines pitch D in the musical scale known as Plato's 'World Soul'⁶. It is also the keynote for the Ancient Greek Phrygian Octave Species, the only one of such scales that is palindromic, in terms of its tones (t) and semitones (tstttst), and also its ratios and its reciprocals. The first seven tones of the descending Phrygian Octave Species, justly tuned, would produce the Mesopotamian heptachord *embubum*, which can then be defined in smaller tone-numbers, starting from D=27. Finally, Professor Amstutz of the University of Heidelberg⁷ is reported to have said: 'Matter's latticed waves are spaced at intervals corresponding to the frets on a harp⁸ or guitar, with analogous sequences of overtones from each fundamental'.

³ Merrick, R., (2009), *Interference: a Grand Scientific Musical Theory*, ISBN 978-0-615-20599-1, p.138, note 70

⁴ In fact, any two numbers treated in the manner of the Fibonacci Series converge on Phi

⁵ One scholar whose work seems to be pointing in the right direction is Hans Hermann Otto in his paper 'Should we pay more attention to the relationship between the golden mean and Archimedes' constant', <https://www.researchgate.net/publication/319063165>

⁶ Crickmore, L. (2009), 'A possible Mesopotamian origin for Plato's World Soul', *Hermathena* 186: 5-13

⁷ Cited in Lawlor, R., (1982), *Sacred Geometry*, Thames and Hudson, p.4

⁸ Presumably this is either a reference to a fretted harp-guitar, or to a zither, or simply just a mistranslation of a word meaning lute

Appendix:

Peter Welsford has kindly offered me his comments on my article. I list below a number of these which seem to be both relevant to and confirmatory of the thesis of the article. Each of the comments, however, is likely to be accorded a different level of seriousness according to the prior assumptions of individual readers.

(a) Merrick's theory of ' Φ damping' has some similarities with the possible closed forms of fscs, up to the seventh dimension, see (f) below.

(b) The number 137 has been the subject of dreams for Pauli (the Nobel Prize winner) with subsequent psychological speculation by Jung concerning his theory of 'synchronicity', and more recently, Leahy, whose dreams have inspired further mathematical work. Pauli died in a hospital room number 137.

(c) According to the Wolfram/Alpha Computational Knowledge Engine, the Golden Angle can be expressed as $(3 - \sqrt{5}) \cdot \pi$, in Rads = 137.50776 This fsc is identical when formulated as 360 (the number of degrees in a circle) divided by the Golden Mean $(\sqrt{5} - 1)/2 = \Phi^2 = 2.618$, a transcendental number. Thanks to Rads $(180/\pi)$ in the first permutation, π becomes the excluded middle, and the circumference x the Diameter of 360 degrees is reconciled with Φ .

(d) The root of Einstein's calculation of the Speed of Light in miles per second gives rise sinusoidally to an fsc = 137.924977. According to Wolfram/Alpha, its possible closed form includes a 7-D Bond of 7 Dimensions in a hypercubic lattice with a Pythagorean Constant.

(e) There exist a large set of fscs, a handful of which percolate the 7th Hyper Dimension, including 82944 (288^2), that represents the centre of a perfect cube, and J. H. Leahy's so-called God particle.

(f) When taken from 180 degrees, the reciprocal cosine of .0765 is 137.1937, i.e. an approximate value for the reciprocal of the Fine Structure Constant. According to Wolfram Alpha (W/A), this bonds into higher hyper dimensions in 'possible

closed forms', and it may therefore be considered as 'a set' of Superstrings in a Superposition State in Quantum Physics.

(g) Michael Sherbon, who has already uploaded onto Academia.edu a number of technical papers about the Fine Structure Constant, Harmonic Proportion and Pythagorean Geometry, has informed the authors that his respect for some of the views concerning 'convergence' of the violinist and climate campaigner, Aubrey Meyer, has increased, since, not unlike this article, they show the convergence of both the Pythagorean Comma and the Fibonacci Sequence onto the Golden Ratio that is linked to the Fine Structure Constant.

(h) It would appear that all the fscs described above meet the musical criteria mentioned in this article.

Leon Crickmore and Peter Welsford,
8 September, 2017.